X(3872) in effective field theory

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Abstract. We consider the implications from the possibility that the recently observed state $X(3872)$ is a meson-antimeson molecule. We write an effective Lagrangian consistent with the heavy-quark and chiral symmetries needed to describe $X(3872)$ and study its properties.

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The unusual properties of $X(3872)$ state, recently discovered in the decay $X(3872) \to J/\psi\pi^+\pi^-$, invited some speculations regarding its possible non-$c\bar{c}$ nature [1]. Since its mass lies tantalizingly close to the $D^{*0}\overline{D}^0$ threshold of 3871.3 MeV, it is tempting to interpret $X(3872)$ as a $D^{*0}\overline{D}^0$ molecule with $J^{PC} = 1^{++}$ quantum numbers [2, 3]. Such molecular states can be studied using techniques of effective field theories (EFT).

This study is possible due to the multitude of scales present in QCD. The extreme smallness of the binding energy, $E_b = (m_{D^0} + m_{D^{0*}}) - M_X = -0.6 \pm 1.1$ MeV, suggests that this state can play the role of the "deuteron" [2] in meson-meson interactions. This fact allows us to use methods similar to those developed for the description of the deuteron, with the added benefit of heavy-quark symmetry. The tiny binding energy of this molecular state introduces an energy scale which is much smaller than the mass of the lightest particle, the pion, whose exchange can provide binding. Then, a suitable effective Lagrangian describing such a system contains only heavy-meson degrees of freedom with interactions approximated by local four-boson terms constrained only by the symmetries of the theory. This approach is similar to the Weinberg's EFT description of the deuteron [4]. While its predictive power is somewhat limited, several model-independent statements can be made. For instance, possible existence of a molecular state in $D^{*0}\overline{D}^0$ channel does not imply a molecular state in the $D^{*0}\overline{D^{*}}^0$ or $D^0\overline{D}^0$ channels.

The general effective Lagrangian consistent with heavy-quark spin and chiral symmetries can be written as [5]

$$\mathscr{L} = \mathscr{L}_2 + \mathscr{L}_4, \tag{1}$$

where the two-body piece that describes the strong interactions of the heavy mesons P and P^* ($P = B, D$) containing one heavy quark Q is well known [6]:

$$\mathscr{L}_2 = -i\mathrm{Tr}\left[\overline{H}^{(Q)} v\cdot D H^{(Q)}\right] - \frac{1}{2m_P}\mathrm{Tr}\left[\overline{H}^{(Q)} D^2 H^{(Q)}\right] + \frac{\lambda_2}{m_P}\mathrm{Tr}\left[\overline{H}^{(Q)}\sigma^{\mu\nu} H^{(Q)}\sigma_{\mu\nu}\right] + \ldots \tag{2}$$

where the ellipsis denotes terms with more derivatives or including explicit factors of light quark masses, or describing pion-H interactions and antimeson degrees of freedom $H_a^{(\overline{Q})}$ and $H_a^{(\overline{Q})\dagger}$. A superfield describing the doublet of pseudoscalar heavy-meson fields $P_a = \left(P^0, P^+\right)$ and their vector counterparts with $v\cdot P_a^{*(Q)} = 0$, is defined as

$H_a^{(Q)} = (1+\not{v})\left[P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right]/2$ (see [6]). The third term in Eq. (2) accounts for the $P-P^*$ mass difference $\Delta \equiv m_{P^*} - m_P = -2\lambda_2/m_P$. The four-body piece is [5]

$$\mathscr{L}_4 = \sum_{i=1}^{2}(-1)^i \frac{C_i}{4}\mathrm{Tr}\left[\overline{H}^{(Q)}H^{(Q)}\Gamma_{i\mu}\right]\mathrm{Tr}\left[H^{(\overline{Q})}\overline{H}^{(\overline{Q})}\Gamma_i^\mu\right] \tag{3}$$

where $\Gamma_{i\mu} = (\gamma_\mu, \gamma_\mu\gamma_5)$. Heavy-quark spin symmetry implies that the same Lagrangian governs the four-boson interactions of *all* $P_a^{(*)} = D^{(*)}$ states. Indeed, not all of these states are bound. Here we shall concentrate on $X(3872)$, which we assume to be a bound state of two *neutral* bosons, $P_a \equiv P^0 \equiv D$ [2]. Evaluating the traces yields for the $D\overline{D^*}$ sector

$$\begin{aligned}\mathscr{L}_{4,DD^*} = &- C_1 D^{(c)\dagger}D^{(c)}D_\mu^{*(\overline{c})\dagger}D^{*(\overline{c})\mu} - C_1 D_\mu^{*(c)\dagger}D^{*(c)\mu}D^{(\overline{c})\dagger}D^{(\overline{c})} \\ &+ C_2 D^{(c)\dagger}D_\mu^{*(c)}D^{*(\overline{c})\dagger\mu}D^{(\overline{c})} + C_2 D_\mu^{*(c)\dagger}D^{(c)}D^{(\overline{c})\dagger}D^{*(\overline{c})\mu} + \ldots\end{aligned} \tag{4}$$

As we show later, the resulting binding energy depends on a *linear combination* of C_1 and C_2. Similarly, one obtains the component Lagrangian governing the interactions of D and $\overline{D}$,

$$\mathscr{L}_{4,DD} = C_1 D^{(c)\dagger}D^{(c)}D^{(\overline{c})\dagger}D^{(\overline{c})}. \tag{5}$$

Clearly, one cannot relate the existence of the bound state in the $D\overline{D^*}$ and $D\overline{D}$ channels, as the properties of the latter will depend only on C_1.

The lowest-energy bound state of D and $\overline{D^*}$ is an eigenstate of charge conjugation,

$$|X_\pm\rangle = \frac{1}{\sqrt{2}}\left[|D^*\overline{D}\rangle \pm |D\overline{D}^*\rangle\right]. \tag{6}$$

To find the bound-state energy of $X(3872)$ with $J^{PC} = 1^{++}$, we shall look for a pole of the transition amplitude $T_{++} = \langle X_+|T|X_+\rangle$. Defining DD^*-DD^* transition amplitudes,

$$\begin{aligned} T_{11} &= \langle D^*\overline{D}|T|D^*\overline{D}\rangle, \quad T_{12} = \langle D^*\overline{D}|T|D\overline{D}^*\rangle, \\ T_{21} &= \langle D\overline{D}^*|T|D^*\overline{D}\rangle, \quad T_{22} = \langle D\overline{D}^*|T|D\overline{D}^*\rangle, \end{aligned} \tag{7}$$

we also have to include a "bubble" resummation of loop contributions, as existence of a bound state is related to a breakdown of perturbative expansion [4]. These amplitudes satisfy a system of Lippmann-Schwinger equations [5]. In an algebraic matrix form,

$$\begin{pmatrix} T_{11} \\ T_{12} \\ T_{21} \\ T_{22} \end{pmatrix} = \begin{pmatrix} -C_1 \\ C_2 \\ C_2 \\ -C_1 \end{pmatrix} + i\widetilde{A}\begin{pmatrix} -C_1 & C_2 & 0 & 0 \\ C_2 & -C_1 & 0 & 0 \\ 0 & 0 & -C_1 & C_2 \\ 0 & 0 & C_2 & -C_1 \end{pmatrix}\begin{pmatrix} T_{11} \\ T_{12} \\ T_{21} \\ T_{22} \end{pmatrix}. \tag{8}$$

The solution of Eq. (8) produces the T_{++} amplitude,

$$T_{++} = \frac{1}{2}\left(T_{11} + T_{12} + T_{21} + T_{22}\right) = \frac{\lambda}{1 - i\lambda\widetilde{A}}, \tag{9}$$

where $\lambda = C_2 - C_1$ and $\widetilde{A}$ is a (divergent) integral

$$\widetilde{A} = \frac{i}{4} 2\mu_{DD^*} \int \frac{d^3q}{(2\pi)^3} \frac{1}{\vec{q}^{\,2} - 2\mu_{DD^*}(E-\Delta) - i\varepsilon} = -\frac{1}{8\pi}\mu_{DD^*}|\vec{p}|\sqrt{1 - \frac{2\mu_{DD^*}\Delta}{\vec{p}^{\,2}}}. \quad (10)$$

Here $E = \vec{p}^{\,2}/2\mu_{DD^*}$, μ_{DD^*} is the reduced mass of the DD^* system. The divergence of the integral of Eq. (10) is removed by renormalization. We chose to define a renormalized λ_R within the *MS* subtraction scheme in dimensional regularization, which does not introduce any new dimensionfull scales into the problem. In this scheme the integral $\widetilde{A}$ is finite, which corresponds to an implicit subtraction of power divergences in Eq. (10). This implies for the transition amplitude

$$T_{++} = \frac{\lambda_R}{1 + (i/8\pi)\lambda_R\,\mu_{DD^*}|\vec{p}|\sqrt{1 - 2\mu_{DD^*}\Delta/\vec{p}^{\,2}}}. \quad (11)$$

The position of the pole of the molecular state on the energy scale can be read off Eq. (11),

$$E_{\rm Pole} = \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3} - \Delta. \quad (12)$$

Recalling the definition of binding energy E_b and that $m_{D^*} = m_D + \Delta$, we infer

$$E_b = \frac{32\pi^2}{\lambda_R^2 \mu_{DD^*}^3}. \quad (13)$$

Assuming E_b = 0.5 MeV, which is one sigma below the central value [1], and the experimental values for the masses, we obtain $\lambda_R \simeq 8.4 \times 10^{-4}$ MeV^{-2}.

Similar considerations apply to $D^0\overline{D}^0$ state, in which case the starting point is the Lagrangian term in Eq. (5). Since it involves only a single term, the calculations are actually easier and involve only one Lippmann-Schwinger equation. The resulting binding energy is then [5] $E_b = (256\pi^2)/(C_{1R}^2 m_D^3)$. Examining this we immediately notice that the existence of a bound state in the $D^*\overline{D}$ channel does not dictate the properties of a possible bound state in, say, $D^0\overline{D}^0$ channel, since C_1 and C_2 are not related to each other. This work was supported in part by the U.S. National Science Foundation under Grant PHY–0244853, and by the U.S. Department of Energy under Contract DE-FG02-96ER41005.

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